Ein bisschen unbestimmt?

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Wenn $$\lim_{x\to x_0} f(x) = \lim_{x\to x_0} g(x) = 0$$

oder $$\lim_{x\to x_0} f(x) = \lim_{x\to x_0} g(x) = \infty$$

dann $$\lim_{x\to x_0} \frac{f(x)}{g(x)} = \lim_{x\to x_0} \frac{f'(x)}{g'(x)}$$

Geht?

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$$\lim_{x\to 0}\frac{\sin x}{x}=\lim_{x\to 0}\frac{\sin' x}{x'}=\lim_{x\to 0}\frac{\cos x}{1}=1$$

$$\lim_{x\to\infty}\left(1+\frac{1}{x}\right)^x=\cdots=\mathrm{e}$$

$$\lim_{x\to 0}\frac{\sin^{42} x}{x^{42}}=\cdots=1$$

Geht nicht?

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$$\lim_{x\to\infty}\frac{x-\sin x}{x+\sin x}=\lim_{x\to\infty}\frac{1-\cos x}{1+\cos x}=?$$

Abschätzung

$$\lim_{x\to\infty}\frac{x-\sin x}{x+\sin x}=\lim_{x\to\infty}\frac{x-\varepsilon}{x+\varepsilon}=\lim_{x\to\infty}\frac{x}{x}=1$$

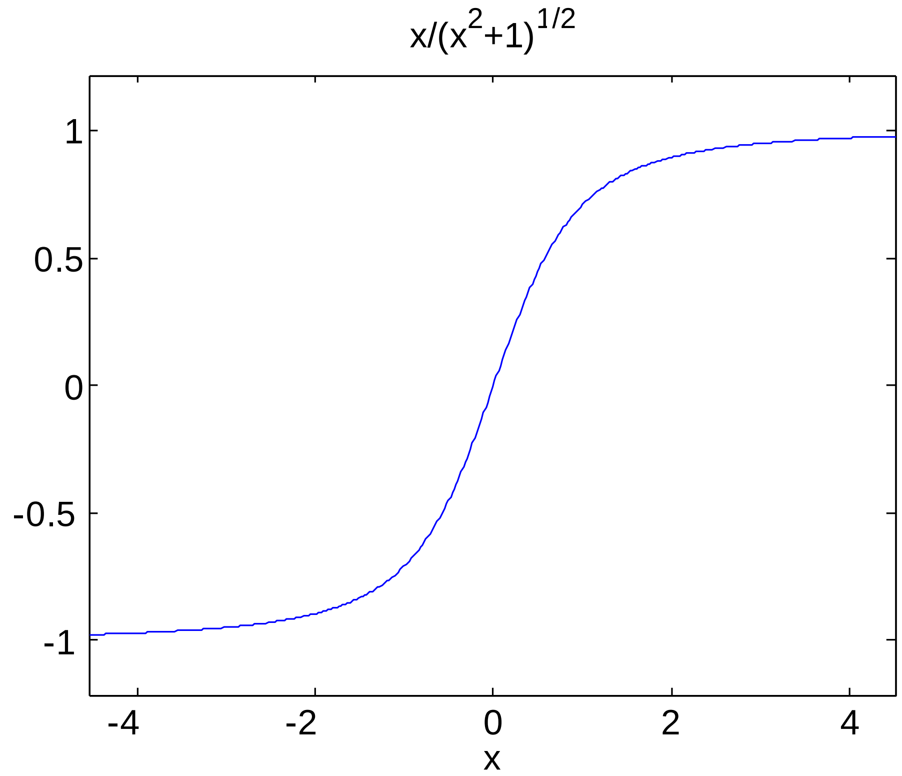
Geht auch nicht?

$$\lim_{x\to\infty}\frac{x}{\left(x^2+1\right)^{\frac{1}{2}}}=\lim_{x\to\infty}\frac{1}{\frac{1}{2}\left(x^2+1\right)^{-\frac{1}{2}}2x}=$$

$$\lim_{x\to\infty}\frac{\left(x^2+1\right)^{\frac{1}{2}}}{x}=\lim_{x\to\infty}\frac{x}{\left(x^2+1\right)^{\frac{1}{2}}}=\cdots?$$

Geht doch!

```
>> syms x
>> f = x/(x^2 + 1)^(1/2)

f =
x/(x^2+1)^(1/2)

>> ezplot (f)
>> subs (f, x, 1e100)

ans =
     1

>> limit (f, x, inf)

ans =
     1
```



Geht immer?

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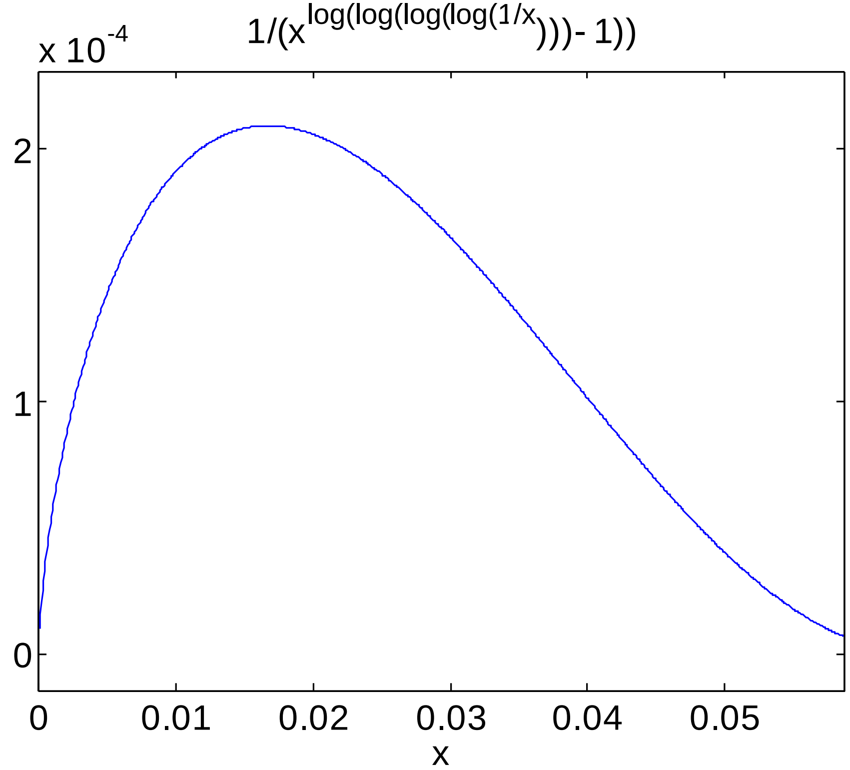
$$\lim_{x\to 0} \frac{1}{x^{\ln\left(\ln\left(\ln\left(\ln\left(\frac{1}{x}\right)\right)\right)\right)-1}}$$




```
>> f = 1/x^(log (log (log (log (1/x)))) - 1);
>> ezplot (f)
```

Einsetzen?

```
>> subs (f, x, 0)
Warning: Divide by zero.

ans =
Inf

>> subs (f, x, 1e-100)

ans =
   4.8704e-048
```

```
>> limit (f, x, 0)

ans =
NaN

>> limit (f, x, 0, 'right')

ans =
Inf

>> solve (diff (f))

ans =
.280843878523400709e-579202
```

Geht manchmal

$$\lim_{x\to\infty}\sqrt{\ln(x+1)}-\sqrt{\ln x}=\lim_{x\to\infty}\frac{\ln(x+1)-\ln x}{\sqrt{\ln(x+1)}+\sqrt{\ln x}}=$$

$$\lim_{x\to\infty}\frac{\ln\left(\frac{x+1}{x}\right)}{\sqrt{\ln(x+1)}+\sqrt{\ln x}}=\lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{\sqrt{\ln(x+1)}+\sqrt{\ln x}}=0$$

Geht häufig

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$$\lim_{x\to 0}\frac{(1+x)^s-1}{x}=\lim_{x\to 0}\frac{1+sx+O\left(x^2\right)-1}{x}=$$

$$\lim_{x\to 0} s+O\left(x\right)=s$$

Was ist eigentlich

$$1^{\infty} = \underbrace{1 \cdot 1 \cdot 1 \cdot \ldots \cdot 1 \cdot 1 \cdot 1}_{\infty} = ?$$

Matlab 7.0.4.365 (R14SP2):

```
>> 1^inf

ans =
   NaN
```

```
>> sym(1)^inf

ans =
1
```

$1^{\infty} = ?$

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Matlab 7.0.4.365	NaN
Maple 9.5	1
Octave 2.1.50	NaN
Reduce 3.6	1
Mathematica 5	NaN
Maxima 5.9.1	1
Derive 6	NaN
MuPAD light 2.5.3	1

$0^0 = ?$

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- Euler, L. (1707-1783): **`0^0 = 1`**
- Cauchy, A. L. (1821): **`0^0 = NaN`**
- Möbius, A. F. (1833): **`0^0 = 1`**

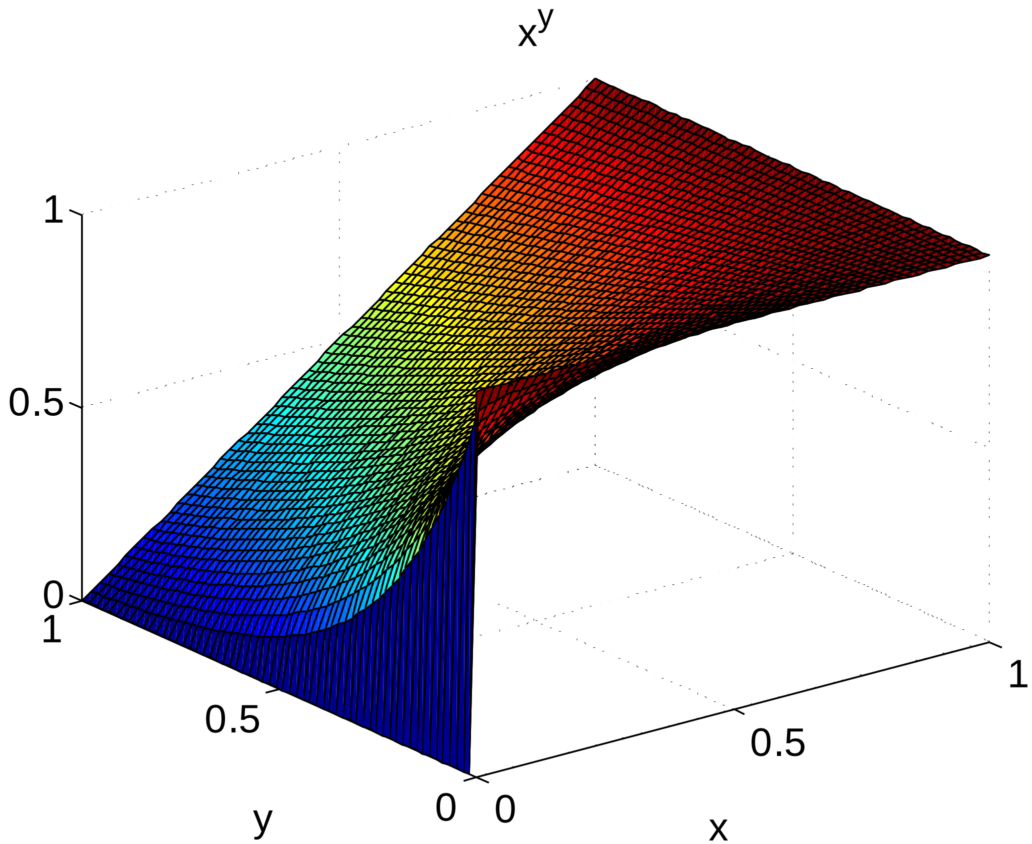
:

- Mathematica (2005): **`0^0 = NaN`**
- Matlab (2005): **`0^0 = 1`**
- Windows XP (2005): **`0^0 = 1`**
- Google (2005): **`0^0 = 1`**

Matlab-Urgestein

Cleve Moler (vs. Buchholz, comp.soft-sys.matlab, 2002)

“However,if
x and y -> 0
at the same
rate, then
the limit of
x^y is 1.“



und außerdem

Binomischer Satz: $$\left(x+y\right)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

mit $x = 0,\ y = n = 1:$ $$\left(0+1\right)^1 = \sum_{k=0}^{1} \binom{1}{k} 0^k 1^{1-k}$$

$$1 = \binom{1}{0} 0^0 1^{1-0} + \binom{1}{1} 0^1 1^{1-1}$$

$$1 = 0^0$$

Trial and ERROR

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Da müht man sich tagelang ab, den Studierenden die Wunderwelt der Grenzwerte durch anschauliche Beispiele nahe zu bringen:

$$\lim_{x \to 8} \frac{1}{x - 8} = \infty$$

Doch wenn man dann eine einfache Transferleistung erhofft:

$$\lim_{x \to 5} \frac{1}{x - 5} = 5$$